Retrieving the impulse response of the Earth due to random electromagnetic forcing

Sharmin Shamsalsadati and Chester J. Weiss*
Department of Geosciences, Virginia Polytechnic Institute and State University, 4044 Derring Hall (0420), Blacksburg, Virginia 24061, USA
(Received 4 March 2009; revised manuscript received 31 December 2009; published 12 March 2010)

A derivation is presented of Green's function in an arbitrary, heterogeneous conductive medium subject to random, ambient, uncorrelated noise sources. The approach for extracting Green's function is based on the correlation of time series of magnetic field components at two independent locations. As in the related case for the electric field, where the volume distribution of noise sources must be spatially correlated with the heterogeneous conductivity distribution, Green's function for magnetic field requires noise sources to be spatially correlated with the volume distribution of magnetic permeability. For applications of electromagnetic imaging of Earth's deep subsurface, the effect of magnetic permeability variations in the subsurface is often assumed to be negligible when compared to the effect of conductivity variations. Hence, the expressions derived here may be useful for passive electromagnetic subsurface imaging, in apparent contrast to their electric field counterparts. Numerical validation exercises are described which validate this theory for Green's function estimation in the low-frequency limit.

DOI: 10.1103/PhysRevE.81.036603 PACS number(s): 41.90.+e, 91.25.Qi, 05.40.Ca, 82.56.Lz

I. INTRODUCTION

The idea of empirical Green's function (EGF) estimation from correlation of time series in response external random forcing has a long history in seismology and acoustics [1] and has recently gained traction toward becoming a mainstream Earth exploration method to image, for example, deep crustal structure [2], hydrocarbon reservoirs [3], and earthquake dynamics [4]. The attraction of such an approach is clear: sources of noise, whose effects previously required heuristic filtering to isolate the primary signal from a known seismic source, could now be embraced in their full complexity, and furthermore, exploited for improved subsurface seismic resolution.

Until recently, EGF theory only appeared applicable to systems whose governing differential equations were invariant under time reversal [5], that is, for nonattenuating systems. However, it has since been shown [6] that for the case of pressure diffusion the EGF could be extracted from correlation of time series as long as the sources of random noise were assumed to be volumetrically distributed throughout the system. Applying this same approach to the quasistatic electromagnetic induction in a heterogeneous electrically conductive material would further require the power spectrum of the noise sources to be spatially correlated with the conductivity variations in the medium if the EGF for electric field is sought [6], but not, as will be shown below, for the case of magnetic field EGF.

Alternatives to the volume-distributed source formulation for electromagnetics have been investigated, e.g., [7–9], in which case the sources are taken to lie on some bounding surface encapsulating the region of study. Unified theories of EGF estimation have also been proposed [10] in which the electric and magnetic fields are inherently coupled. However, to our knowledge, the present work is the first explicit demonstration of EGF estimation for decoupled magnetic fields and heterogeneous media assuming the area of study is fully impregnated with random point sources of current or time varying magnetization.

Take Faraday's law of induction,

$$\nabla \times \mathbf{E} = -\partial_t \mu(\mathbf{H} + \mathbf{M_s}) \tag{1}$$

and Ampere's current law

$$\nabla \times \mathbf{H} = \sigma \mathbf{E} + \partial_t \varepsilon \mathbf{E} + \mathbf{J_s} \tag{2}$$

as the starting point for the discussion that follows, where the electric field $\mathbf{E}$, magnetic field $\mathbf{H}$ are functions of both time t and position $\mathbf{r}$ throughout a stationary medium of heterogeneous electrical conductivity $\sigma(\mathbf{r})$, dielectric permittivity $\varepsilon(\mathbf{r})$, and magnetic permeability $\mu(\mathbf{r})$, and subject to both electric $\mathbf{J_s}$ and magnetic $\mathbf{M_s}$ sources. Assuming the Fourier Transform with respect to time of Eqs. (1) and (2) exists such that $\partial_t \to i\omega$, that is

$$\mathfrak{F}\{\mathbf{F}(\mathbf{r},t)\} = \mathbf{f}(\mathbf{r},\omega) = \int_{-\infty}^{\infty} \mathbf{F}(\mathbf{r},t)\exp(-i\omega t)dt$$

with $i=\sqrt{-1}$, the electric and magnetic fields are mapped into the complex frequency domain as $(\mathbf{E},\mathbf{H}) \to (\mathbf{e},\mathbf{h})$. Provided the electrical conductivity and magnetic permeability are time-invariant, the frequency-domain equivalents of Eqs. (1) and (2) become

$$\nabla \times \mathbf{e} = -i\omega\mu(\mathbf{h} + \mathfrak{F}\{\mathbf{M_s}\}) \tag{3}$$

and

$$\nabla \times \mathbf{h} = \hat{\sigma}\mathbf{e} + \mathfrak{F}\{\mathbf{J_s}\} \tag{4}$$

with complex conductivity $\hat{\sigma}(\mathbf{r}) = \sigma(\mathbf{r}) + i\omega\varepsilon(\mathbf{r})$. Substituting Eq. (3) into Eq. (4) results in the fundamental partial differential equation from which Green's function will ultimately be derived

*cjweiss@vt.edu

$$\nabla \times \frac{1}{\hat{\sigma}} \nabla \times \mathbf{h} + i\omega\mu\mathbf{h} = \mathbf{f}, \tag{5}$$

where

$$\mathbf{f}(\mathbf{r},\omega) = \mathfrak{F}\left\{ - i\omega\mu\mathbf{M_s}(\mathbf{r},t) + \nabla \times \frac{1}{\hat{\sigma}}\mathbf{J_s}(\mathbf{r},t)\right\}. \tag{6}$$

Observe that Eq. (5) is a "full physics" solution to the Maxwell equations where the spatially variable complex conductivity $\hat{\sigma}$ permits both diffusive and wavelike energy transport by its real and imaginary components, respectively. In regions such as the air, where the electrical conductivity is effectively zero, the value of $\hat{\sigma}$ is dominated by its nonzero imaginary component, and hence, the quotient in curl-curl term of Eq. (5) remains well-defined. Denoting complex conjugation by the superscript *, the time-reversed equivalent of Eq. (5) is

$$\nabla \times \frac{\hat{\sigma}}{|\hat{\sigma}|^2} \nabla \times \mathbf{h}^* - i\omega\mu\mathbf{h}^* = \mathbf{f}^*. \tag{7}$$

In Eq. (3)–(5) and (7) note that the material properties σ, ε, and μ remain space-dependent (scalar) quantities, representing an arbitrary (isotropic) heterogeneous medium, with an implicit dependence on the position vector $\mathbf{r}$ unless stated otherwise. This more compact notation will be continued throughout the remainder of the manuscript without any loss of generality.

II. REPRESENTATION THEOREMS OF THE CORRELATION AND CONVOLUTION TYPE

Following the development in Snieder [6], which itself is based on Fokkema and van den Berg and co-workers [11,12], representation theorems of the correlation and convolution type are derived specifically for the time-forward and time-reverse expressions in Eq. (5) and (7). For each of the representation theorems, the magnetic field resulting from an arbitrary source $\mathbf{f}_A$ is denoted with a subscript as $\mathbf{h}_A$, and similarly for a source B.

Considering first the time-forward expression in Eq. (5), the fields $\mathbf{h}_A$ and $\mathbf{h}_B$ are related by

$$\mathbf{h}_B \cdot \nabla \times \frac{1}{\hat{\sigma}} \nabla \times \mathbf{h}_A + i\omega\mu(\mathbf{h}_B \cdot \mathbf{h}_A) = \mathbf{h}_B \cdot \mathbf{f}_A \tag{8}$$

and

$$\mathbf{h}_A \cdot \nabla \times \frac{1}{\hat{\sigma}} \nabla \times \mathbf{h}_B + i\omega\mu(\mathbf{h}_A \cdot \mathbf{h}_B) = \mathbf{h}_A \cdot \mathbf{f}_B \tag{9}$$

for arbitrary sources A and B, where $\cdot$ denotes the dot product. Subtraction of Eq. (9) from Eq. (8) and subsequent volume integration over the domain Ω results in

$$\int_\Omega \left(\mathbf{h}_B \cdot \nabla \times \frac{1}{\hat{\sigma}} \nabla \times \mathbf{h}_A - \mathbf{h}_A \cdot \nabla \times \frac{1}{\hat{\sigma}} \nabla \times \mathbf{h}_B \right) d\Omega = \int_\Omega (\mathbf{h}_B \cdot \mathbf{f}_A - \mathbf{h}_A \cdot \mathbf{f}_B) d\Omega, \tag{10}$$

the left-hand side of which is equivalent to a surface integral over the domain Γ bounding Ω with outward-pointing normal $\mathbf{n}$ as

$$\int_\Gamma \frac{\hat{\sigma}^*}{|\hat{\sigma}|^2} [(\nabla \times \mathbf{h}_A) \times \mathbf{h}_B - (\nabla \times \mathbf{h}_B) \times \mathbf{h}_A] \cdot \mathbf{n} \; d\Gamma. \tag{11}$$

For any compact sources A and B, this integral vanishes as $\Omega = (-\infty, +\infty)^3$ due to exponentially vanishing fields $\mathbf{h}_A$ and $\mathbf{h}_B$ on Γ, and hence, the *representation theorem of convolution type* is given by

$$\int_\Omega (\mathbf{h}_B \cdot \mathbf{f}_A - \mathbf{h}_A \cdot \mathbf{f}_B) d\Omega = 0. \tag{12}$$

Note, however, that the assumption of an infinite domain Ω is not a prerequisite for Eq. (12) to hold. All domains Ω and source pairs (A,B) where Eq. (11) is equal to zero are equally valid and yield an equivalent representation theorem Eq. (12). However, for simplicity, only the infinite domain will be considered further here.

Following in a similar fashion to the preceding development of Eq. (8) through Eq. (12), consideration of the time-reversed expression Eq. (7) for source B yields the following pair of coupled equations:

$$\mathbf{h}_B^* \cdot \nabla \times \frac{\hat{\sigma}^*}{|\hat{\sigma}|^2} \nabla \times \mathbf{h}_A + i\omega\mu(\mathbf{h}_B^* \cdot \mathbf{h}_A) = \mathbf{h}_B^* \cdot \mathbf{f}_A \tag{13}$$

and

$$\mathbf{h}_A \cdot \nabla \times \frac{\hat{\sigma}}{|\hat{\sigma}|^2} \nabla \times \mathbf{h}_B^* - i\omega\mu(\mathbf{h}_A \cdot \mathbf{h}_B^*) = \mathbf{h}_A \cdot \mathbf{f}_B^*. \tag{14}$$

Subtraction of Eq. (14) from Eq. (13) and subsequent volume integration over the domain Ω yields the *representation theorem of the correlation type*,

$$-2\int_\Omega \frac{\mathrm{Im}[\hat{\sigma}]}{|\hat{\sigma}|^2} (\nabla \times \mathbf{h}_A) \cdot (\nabla \times \mathbf{h}_B^*) d\Omega + 2i\omega \int_\Omega \mu \mathbf{h}_B^* \cdot \mathbf{h}_A d\Omega = \int_\Omega (\mathbf{h}_B^* \cdot \mathbf{f}_A - \mathbf{h}_A \cdot \mathbf{f}_B^*) d\Omega, \tag{15}$$

provided the following boundary integral is zero:

$$\int_\Gamma \left[\frac{\hat{\sigma}^*}{|\hat{\sigma}|^2} (\nabla \times \mathbf{h}_A) \times \mathbf{h}_B^* - \frac{\hat{\sigma}}{|\hat{\sigma}|^2} (\nabla \times \mathbf{h}_B^*) \times \mathbf{h}_A \right] \cdot \mathbf{n} \; d\Gamma. \tag{16}$$

As was noted for Eq. (11), there may be several $(A,B)-\Omega$ configurations that result in Eq. (16) equaling zero. For now, the simple requirement that (A,B) are compact sources within the infinite domain Ω simultaneously sets to zero the value of the integrals in Eq. (11) and Eq. (16).

Furthermore, we observe that in the quasistatic limit $(\omega\varepsilon/\sigma \ll 1)$ used for low-frequency electromagnetic induction studies of the deep Earth, with the additional assumption that the spatial variability in the magnetic permeability of the rocks in question is minimal and therefore assumed to take the value of free space $\mu_0 = 4\pi \times 10^{-7}$ H/m, the representation theorem of correlation type simplifies considerably,

$$2i\omega\mu_0\int_\Omega \mathbf{h}_B^*\cdot\mathbf{h}_A d\Omega \approx \int_\Omega (\mathbf{h}_B^*\cdot\mathbf{f}_A - \mathbf{h}_A\cdot\mathbf{f}_B^*)\ d\Omega \quad (17)$$

and is reminiscent of the functional form for the scalar pressure diffusion equation [6].

III. REPRESENTATION THEOREMS AND GREEN'S FUNCTIONS

We denote by $\bar{\mathbf{h}}_A$ and $\bar{\mathbf{h}}_B$, respectively, Green's functions for magnetic field due to the impulsive sources $\mathbf{f}_A=\mathbf{s}_A\delta(\mathbf{r}-\mathbf{r}_A)$ and $\mathbf{f}_B=\mathbf{s}_B\delta(\mathbf{r}-\mathbf{r}_B)$ where $\delta(\cdot)$ is Dirac's delta function and $\mathbf{s}$ is a unit vector. Hence, the representation theorem of convolution type in Eq. (12) reduces to a reciprocity relation

$$\mathbf{s}_A\cdot\bar{\mathbf{h}}_B(\mathbf{r}_A) - \mathbf{s}_B\cdot\bar{\mathbf{h}}_A(\mathbf{r}_B) = 0 \quad (18)$$

with the usual caveat that the integral Eq. (11) vanishes. Moreover, these specific definition for the source terms $\mathbf{f}_A$ and $\mathbf{f}_B$ simplify Eq. (17) to,

$$\mathbf{s}_A\cdot\bar{\mathbf{h}}_B^*(\mathbf{r}_A) - \mathbf{s}_B\cdot\bar{\mathbf{h}}_A(\mathbf{r}_B) = 2i\omega\mu_0\int_\Omega \bar{\mathbf{h}}_B^*\cdot\bar{\mathbf{h}}_A d\Omega, \quad (19)$$

while the additional constraints of the quasistatic limit, constant magnetic permeability, and vanishing boundary integral Eq. (16) remain. Using the reciprocity expression Eq. (18), the left-hand side of Eq. (19) can now be written entirely in terms of Green's function observed at B for a source at A

$$\mathbf{s}_A\cdot[\bar{\mathbf{h}}_B^*(\mathbf{r}_A) - \bar{\mathbf{h}}_B(\mathbf{r}_A)] = 2i\omega\mu_0\int_\Omega \bar{\mathbf{h}}_B^*\cdot\bar{\mathbf{h}}_A d\Omega \quad (20)$$

and vice versa

$$\mathbf{s}_B\cdot[\bar{\mathbf{h}}_A^*(\mathbf{r}_B) - \bar{\mathbf{h}}_A(\mathbf{r}_B)] = 2i\omega\mu_0\int_\Omega \bar{\mathbf{h}}_B^*\cdot\bar{\mathbf{h}}_A d\Omega. \quad (21)$$

To understand the role of random noise in the evaluation of the volume integrals on the right side of Eq. (20) and (21), let's start by examining this term in its discrete form where the domain Ω is discretized into a set of infinitesimal differential elements $d\Omega_j$ such that

$$\int_\Omega \bar{\mathbf{h}}_B^*\cdot\bar{\mathbf{h}}_A d\Omega \approx \sum_j \bar{\mathbf{h}}_B^*(\mathbf{r}_j)\cdot\bar{\mathbf{h}}_A(\mathbf{r}_j)d\Omega_j. \quad (22)$$

Letting $\mathbf{s}_A=\mathbf{s}_B=\mathbf{s}$, substitution of the reciprocity relation Eq. (18) into Eq. (22) yields

$$\bar{\mathbf{h}}_B^*(\mathbf{r}_j)\cdot\bar{\mathbf{h}}_A(\mathbf{r}_j) = (\mathbf{s}\cdot\bar{\mathbf{h}}_j^*(\mathbf{r}_B))(\mathbf{s}\cdot\bar{\mathbf{h}}_j(\mathbf{r}_A)), \quad (23)$$

where the sources at point $\mathbf{r}_j$ are also taken in to lie in the $\mathbf{s}$ direction. It is now clear that Green's functions in Eq. (20) and (21), or more specifically, the $\mathbf{s}$ component of which, arise from an the summation of an infinite number of $\mathbf{s}$-directed impulsive sources j throughout the volume Ω, whose effects need only be known at two locations: $\mathbf{r}_B$ and $\mathbf{r}_A$.

To account for the variable spectral content $f(\omega)$ of a naturally occurring random noise source, the product of the squared power spectrum amplitude $|f(\omega)|^2$ with the j-summation terms on the right-hand side of Eq. (23) is $|f(\omega)|^2(\mathbf{s}\cdot\bar{\mathbf{h}}_j^*(\mathbf{r}_B))(\mathbf{s}\cdot\bar{\mathbf{h}}_j(\mathbf{r}_A))$, which, in turn, can be written as

$$\left(\int \mathbf{s}\cdot\bar{\mathbf{h}}_j(\mathbf{r})f(\omega)\delta(\mathbf{r}-\mathbf{r}_B)d\Omega\right)^* \times\left(\int \mathbf{s}\cdot\bar{\mathbf{h}}_j(\mathbf{r})f(\omega)\delta(\mathbf{r}-\mathbf{r}_A)d\Omega\right). \quad (24)$$

As a consequence of the representation theorem of convolution type, Eq. (12), the expression in Eq. (24) simplifies to $(\mathbf{s}\cdot\mathbf{h}_j(\mathbf{r}_B))^*(\mathbf{s}\cdot\mathbf{h}_j(\mathbf{r}_A))$, whereupon substitution into Eq. (20) and (21), our final expression for Green's function in the Fourier domain emerges

$$\mathbf{s}\cdot[\bar{\mathbf{h}}_B^*(\mathbf{r}_A) - \bar{\mathbf{h}}_B(\mathbf{r}_A)]|f(\omega)|^2 = \mathbf{s}\cdot[\bar{\mathbf{h}}_A^*(\mathbf{r}_B) - \bar{\mathbf{h}}_A(\mathbf{r}_B)]|f(\omega)|^2 = 2i\omega\mu_0\langle(\mathbf{s}\cdot\mathbf{h}(\mathbf{r}_B))^*(\mathbf{s}\cdot\mathbf{h}(\mathbf{r}_A))\rangle, \quad (25)$$

where $\langle\cdot\rangle$ denotes expectation value.

Inspection of Eq. (25) reveals that the difference between Green's function for magnetic field and its complex conjugate projected in the $\mathbf{s}$ direction and scaled by the power spectrum of the uncorrelated and volume-distributed random noise sources, is given simply by the correlation of the $\mathbf{s}$ components of the field measured at two distinct points. Transforming this expression into the time domain, the product between spectral power density and Green's function becomes a convolution operation, whereas the prefactor $i\omega$ maps into a time-derivative. Hence, in the time domain, Green's function for magnetic field $\bar{\mathbf{H}}(\mathbf{r},t)$ becomes

$$\begin{aligned}\mathbf{s}\cdot[\bar{\mathbf{H}}_B(\mathbf{r}_A,-t) - \bar{\mathbf{H}}_B(\mathbf{r}_A,t)] * F(t) &= \mathbf{s}\cdot[\bar{\mathbf{H}}_A(\mathbf{r}_B,-t) - \bar{\mathbf{H}}_A(\mathbf{r}_B,t)] * F(t)\\ &= 2\mu_0\frac{\partial}{\partial t}\langle[\mathbf{s}\cdot\mathbf{H}(\mathbf{r}_B,t)]\otimes[\mathbf{s}\cdot\mathbf{H}(\mathbf{r}_A,t)]\rangle, \end{aligned}\quad (26)$$

in which $F(t)$ is the autocorrelation of the noise. The symbols $\otimes$ and $*$ denote correlation and convolution, respectively.

IV. VALIDATION EXAMPLE

To determine whether the aforementioned theory holds promise for eventual interpretation of observational data, a simple numerical experiment was conducted to validate the accuracy of Eq. (25) in the low frequency limit upon which the theory is based. The calculation is as follows: Assuming a double-half space model with conductivity $\sigma=1.0$ S/m in the $z>0$ region and $\sigma=0.1$ S/m elsewhere, we compute the EGF at two points A and B due to a volume distribution of 100 Hz "noise" sources whose response is recorded at these two points. For simplicity, we choose $\mathbf{s}=\hat{z}$, and further distribute the noise sources equidistant over a $\Delta=25$ m grid in the x, y, and z directions. With this model configuration the corresponding skin depth is 160 m in the resistive half space and 50 m in the conductive one. Hence, with points A and B at coordinates $(x,y,z)_A=(206.25,206.25,-6.25)$ m and $(x,y,z)_B=(-206.25,-206.25,-6.25)$ m, respectively, a

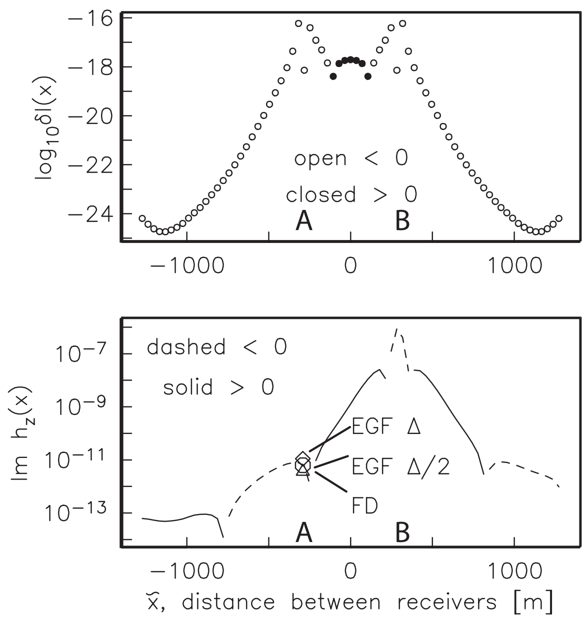

FIG. 1. (top) Relative contribution of noise sources δI to EGF estimation as a function of position $\tilde{x}$ along a line passing through two passive receiver locations A and B at $\tilde{x}=\pm 291$ m. (bottom) In symbols, finite difference (FD) and EGF calculated values of the vertical magnetic field at A due to a source $\mathbf{M}_s=\hat{z}\delta(\mathbf{r}-\mathbf{r}_B)$. Lines indicate FD-computed h_z at $z=0$.

noise source volume extending from $|x|,|y|,|z|\leq 900$ m encompasses several skin depths of distance between observation points A and B and the noise sources.

Choosing $\Delta=25$ m results in 389 017 independent noise-source forward calculations. Halving Δ increases this number to over three million. Hence, even though quasianalytic solutions for induction in layered media have been known and revisited for over a century [13–17], the Hankel transforms at the core of such calculations amount to a significant computational cost when the number of forward calculations is large, as in the present case. Various quadrature and digital filter techniques have been developed to minimize this cost [18–20], but even at 10–100 calculations per second, the wall clock time required for the proposed noise volume quickly escalates to several hours for a desktop computer.

To make this straightforward problem tractable in a reasonable amount of time, we turn instead to the staggered grid finite difference (FD) method [21,22], which by virtue of the reciprocity relation in Eq. (18), allows for the calculation of millions of noise sources with only two forward solves: One with a source at A, the other with a source at B. For the calculations shown here, the finite difference grid is partitioned over a modest $81\times 81\times 81$ nodes on a $1\times 1\times 1$ km volume.

The numerical results (Fig. 1, bottom) are in general agreement with the direct-calculated FD field value at point A and the EGF-estimated ones (symbols), with increasing accuracy realized by the dense $\Delta/2$ distribution of noise sources. FD-computed field values $h_z(\tilde{x})$ along the $z=0$ elevation (lines) illustrate the exponential decay and oscillatory nature of the magnetic field as a function of distance. Notice that the point A lies near a rapid sign-inflexion in the field and therefore represents a particularly challenging place to recover the EGF. To assess which of noise sources contribute most to the EGF estimation (Fig. 1, top) we plot the integrand in the left-hand side of Eq. (17) along the same $z=0$ elevation. The exponential decay and oscillatory behavior of the integrand along this profile reinforce our appreciation for the numerical difficulty of accurately evaluating this multidimensional integral.

V. DISCUSSION

The magnetotelluric (MT) method is a classic geophysical method for inferring the spatial distribution of electrical conductivity of Earth's interior which relies on ambient electromagnetic disturbances of magneto- and ionosphere origin for the source of electromagnetic forcing [23,24]. Inaccuracies in the resulting inferences on the physiochemical state of Earth's interior can be amplified by nonplane wave sources such as power lines, electric fences, trains and the like. Contrary to the MT method, the EGF procedure outlined above *relies* on random uncorrelated noise sources and, hence, has the potential for improved subsurface mapping in areas where the MT method fails due to excessive cultural interference.

Previous work on dissipative, scalar fields (e.g., pressure diffusion) has demonstrated that time-reversal invariance of the governing differential equation is not a prerequisite for EGF estimation [6]. The work presented here parallels the development of the pressure diffusion problem and applies the analysis to low-frequency electromagnetism, treating the fields in their full vector form. Like the pressure diffusion case, the low-frequency magnetic field EGF can be recovered by cross-correlation of time series measured at two discrete locations and time-reversal invariance is not a prerequisite.

The assumption of low-frequencies—that is, the quasistatic limit—is only a simplifying component of the preceding development and does not affect the implications of the final result in Eq. (25). To see what effect incorporation of "full physics" would have, observe that the first integral in Eq. (15), when retained to account for the wave propagation terms, can be rewritten as $i\omega\int_\Omega \varepsilon\mathbf{e}_A\cdot\mathbf{e}_B^* d\Omega$. Assuming that electric permittivity is sufficiently uniform throughout the volume Ω, this added term would ultimately result in the additional requirement of correlating electric fields, too. Regardless of whether the electric field terms are kept or discarded, the magnetic field EGF is a direct reflection of the conductivity distribution throughout the system. Our work shows that *a priori* knowledge of this distribution is not required for EGF estimation.

It has been previously pointed out that when considering the electromagnetic problem, the noise sources must be spatially correlated with the conductivity variations in the medium [6]. For the case of magnetic fields, we have shown that this restriction is no longer necessary. However, when considering electric field Green's functions, its necessity is clear: The conductivity term in the electric field "curl-curl" equation plays the same functional role as the permeability does in Eq. (5). And while it's reasonable in many geophysical exploration scenarios to neglect permeability variations in the rocks, a corresponding dismissal of the conductivity variations would be defensible only in exceptional circumstances [25].

Examination of the final expression in Eqs. (25) and (26) for the superposition of the magnetic field EGF and its time-reversed form reveals that parallel components (**s**-directed) of the ambient magnetic field are required at the two locations A and B, *and* that the noise sources are all taken to be polarized in the same **s** direction. While the former is unremarkable from an implementation perspective, the latter appears absurd in light of the random nature of the noise. To reconcile this apparent problem, we observe that a random spatial distribution of point noise sources might also be randomly distributed in its polarization, and hence, within this latter distribution there will be a component parallel to **s** for each of the sources.

The preceding development also does not place any restrictions on the type of noise, other than is must be "infinitesimal" in some spatial sense. Both magnetic and electric current sources are admissible, including idealized dipole sources. Hence, electromagnetically cluttered environments such as urban areas, oilfields and industrial facilities may provide a rich spectrum of noise from which to draw upon for Green's function estimation and are attractive sites for future potential validation exercises with observed time series of magnetic field. The effect of *correlated*, heterogeneous, and anisotropic noise sources on the recovered EGF is an interesting question, but lies beyond the scope of the present study and will be addressed in future publications.

ACKNOWLEDGMENT

Special thanks are extended to Roel Snieder for helpful comments and clarifying our thinking on the ideas presented here.

[1] J. Claerbout, Geophysics **33**, 264 (1968).
[2] G. D. Bensen, M. H. Ritzwoller, and N. M. Shapiro, J. Geophys. Res. **113**, B05306 (2008).
[3] A. Bakulin and R. Calvert, Geophysics **71**, SI139 (2006).
[4] S. de Lorenzo, M. Filippucci, and E. Boschi, J. Geophys. Res. **113**, B10314 (2008).
[5] A. Curtis, P. Gerstoft, H. Sato, R. Snieder, and K. Wapenaar, The Leading Edge **25**, 1082 (2006).
[6] R. Snieder, Phys. Rev. E **74**, 046620 (2006).
[7] E. Slob and K. Wapenaar, Geophys. Res. Lett. **34**, L05307 (2007).
[8] E. Slob and K. Wapenaar, IEEE Trans. Geosci. Remote Sens. **45**, 2501 (2007).
[9] K. Wapenaar, E. Slob, and R. Snieder, Geophys. Prospect. **56**, 419 (2008).
[10] K. Wapenaar, E. Slob, and R. Snieder, Phys. Rev. Lett. **97**, 234301 (2006).
[11] J. T. Fokkema and P. M. van den Berg, *Seismic Applications of Acoustic Reciprocity* (Elsevier, Amsterdam, 1993).
[12] J. T. Fokkema and P. M. van den Berg, in *Wavefields and Reciprocity*, edited by P. M. van den Berg, H. Blok, and J. T. Fokkema (Delft University Press, Delft, 1996), pp. 99–108.
[13] H. Weyl, Ann. Phys. **60**, 481 (1919).
[14] A. Sommerfeld, Ann. Phys. **28**, 665 (1909).
[15] A. D. Chave and C. S. Cox, J. Geophys. Res. **87**, 5327 (1982).
[16] J. R. Wait, Can. J. Phys. **39**, 1017 (1961).
[17] A. Baños, *Dipole Radiation in the Presence of a Conducting Halfspace* (Pergamon Press, New York, 1966).
[18] A. D. Chave, Geophysics **48**, 1671 (1983).
[19] D. Guptasarma and B. Singh, Geophys. Prospect. **45**, 745 (1997).
[20] F. N. Kong, Geophys. Prospect. **55**, 83 (2007).
[21] C. J. Weiss and S. Constable, Geophysics **71**, G321 (2006).
[22] K. Yee, IEEE Trans. Antennas Propag. **14**, 302 (1966).
[23] A. N. Tikhonov, Dokl. Akad. Nauk **73**, 295 (1950).
[24] L. Cagniard, Geophysics **18**, 605 (1953).
[25] T. Meglish, Y. Li, L. Pasion, D. Oldenburg, R. L. van Dam, and S. Billings, *SEG Technical Program Expanded Abstracts* (Society for Exploration Geophysics, Tulsa, OK, 2008), Vol. 27, pp. 1248–1251.